

# *THE PHYSICS OF FOAM*

- **Boulder School for Condensed Matter and Materials Physics**  
**July 1-26, 2002: Physics of Soft Condensed Matter**

## **1. Introduction**

Formation

Microscopics

## **2. Structure**

Experiment

Simulation

## **3. Stability**

Coarsening

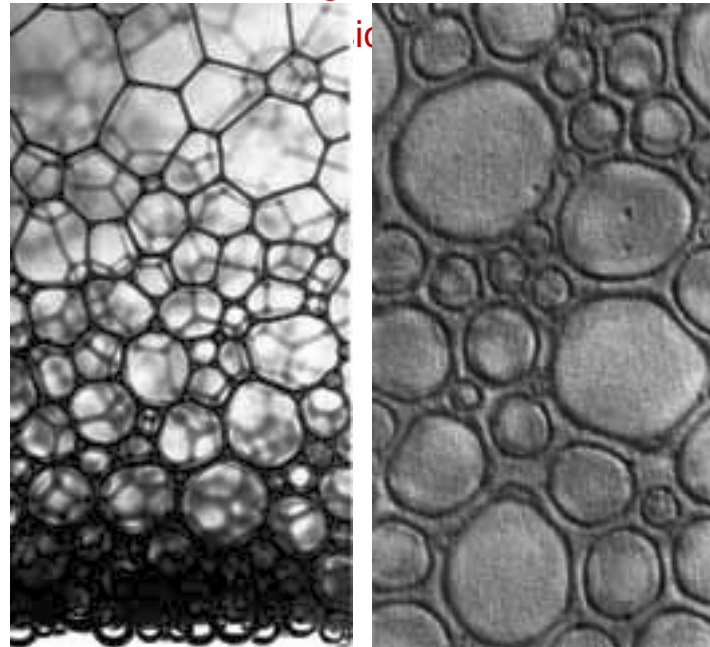
Drainage

## **4. Rheology**

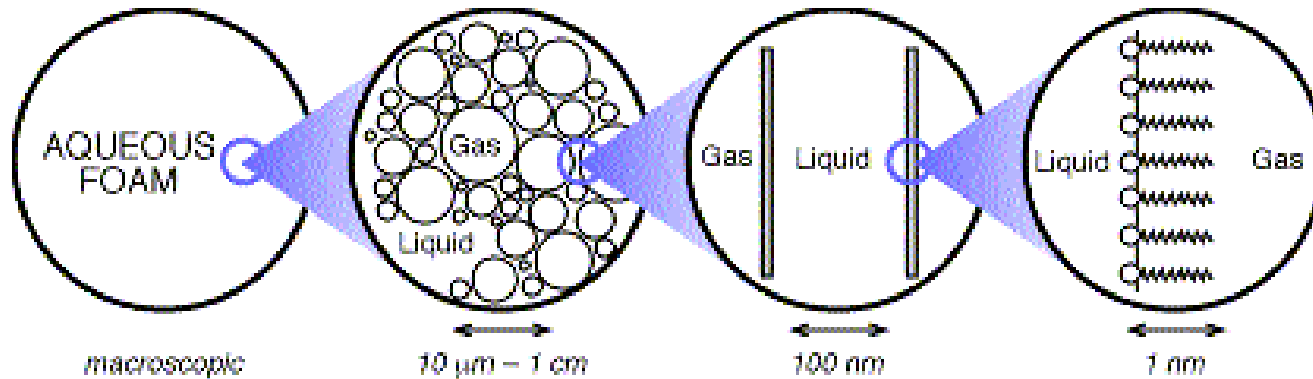
Linear response

Rearrangement & flow

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1547*



- ...a random packing of bubbles in a relatively small amount of liquid containing surface-active impurities
  - Four levels of structure:



- Three means of time evolution:

- Gravitational drainage
- Film rupture
- Coarsening (gas diffusion from smaller to larger bubbles)

- ...a most unusual form of condensed matter
  - Like a gas:
    - volume  $\sim$  temperature / pressure
  - Like a liquid:
    - Flow without breaking
    - Fill any shape vessel
      - Under large force, bubbles rearrange their packing configuration
  - Like a solid:
    - Support small shear forces elastically
      - Under small force, bubbles distort but don't rearrange

- **Everyday life:**
  - detergents
  - foods (ice cream, meringue, beer, cappuccino, ...)
  - cosmetics (shampoo, mousse, shaving cream, tooth paste, ...)
- **Unique applications:**
  - firefighting
  - isolating toxic materials
  - physical and chemical separations
  - oil recovery
  - cellular solids
- **Undesirable occurrences:**
  - mechanical agitation of multicomponent liquid
  - pulp and paper industry
  - paint and coating industry
  - textile industry
  - leather industry
  - adhesives industry
  - polymer industry
  - food processing (sugar, yeast, potatoes)
  - metal treatment
  - waste water treatment
  - polluted natural waters

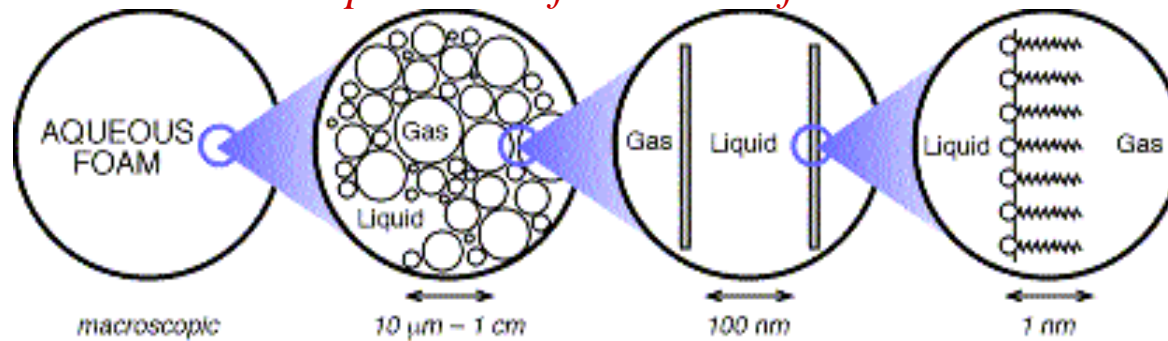
- familiar!
- important!

- *need to control stability and mechanics*
- *must first understand microscopic structure and dynamics...*

# Condensed-matter challenge

- To understand the stability and mechanics of bulk foams in terms of the behavior at microscopic scales

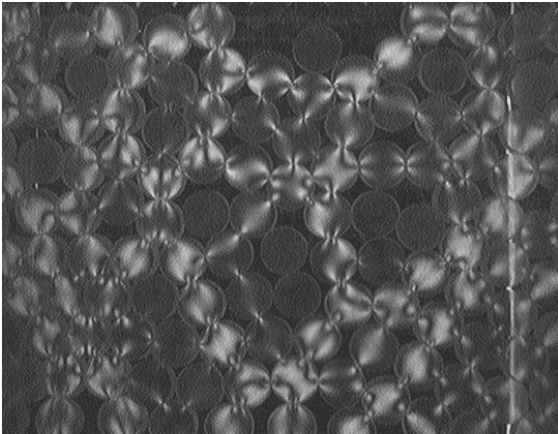
*bubbles are the “particles” from which foams are assembled*



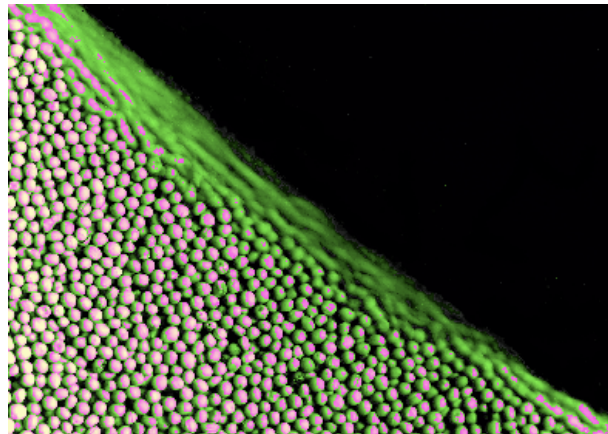
- Easy to relate surfactant-film and film-bubble behaviors
- Hard to relate bubble-macro behavior
  - Opaque: *no simple way to image structure*
  - Disordered: *no periodicity*
  - $k_B T \ll$  interaction energy: *no stat-mech.*
  - Flow beyond threshold: *no linear response*

• hard problems!  
• new physics!

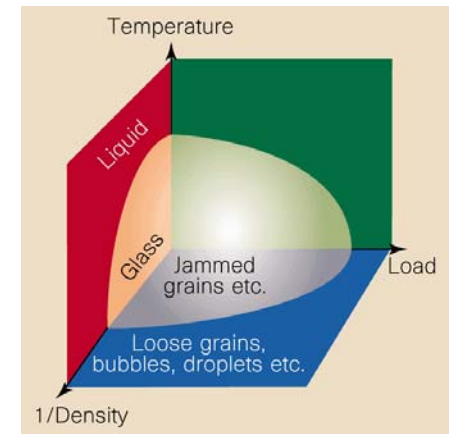
- Similar challenge for seemingly unrelated systems
  - Tightly packed collections of bubbles, droplets, grains, cells, colloids, fuzzy molecules, tectonic plates,....
    - jammed/solid-like: *small-force / low-temperature / high-density*
    - fluid/liquid-like: *large-force / high-temperature / low-density*



force-chains (S. Franklin)




avalanches (S.R. Nagel)



universality?

- visit the websites of these Summer 2002 conferences to see examples of current research on aqueous foams
  - Gordon Research Conference on Complex Fluids
    - Oxford, UK
  - EuroFoam 2002
    - Manchester, UK
  - Foams and Minimal Surfaces
    - Isaac Newton Institute for Mathematical Sciences
  - Geometry and Mechanics of Structured Materials
    - Max Planck Institute for the Physics of Complex Systems

 after these lectures, you should be in a good position to understand the issues being addressed & progress being made!

# General references

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18. S.A. Koehler, S. Hilgenfeldt, and H.A. Stone, "A generalized view of foam drainage," *Langmuir* **16**, 6327-6341 (2000).
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20. J. Banhart and D. Weaire, "On the road again: Metal foams find favor," *Physics Today* **55**, 37-42 (July 2002).

Thanks for the images:

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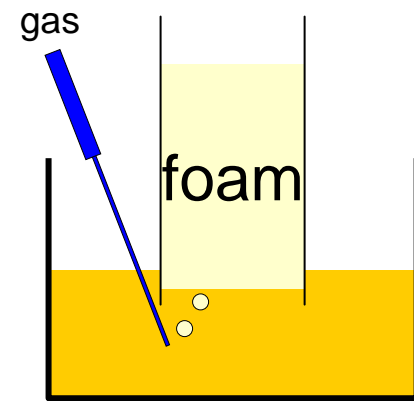
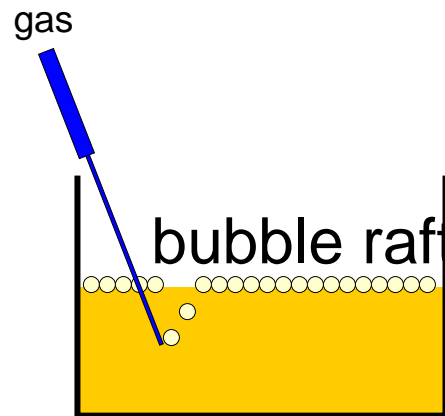
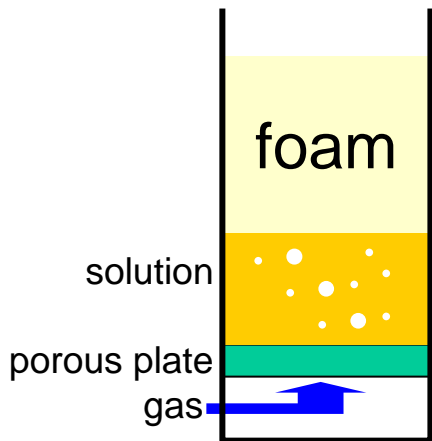
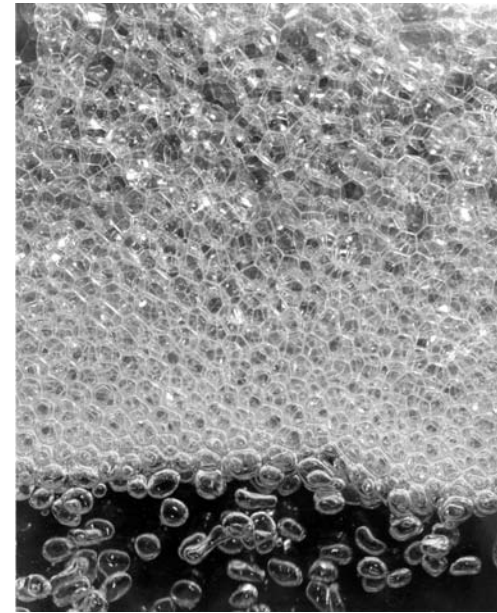
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# Foam production I.

- Shake, blend, stir, agitate, etc.
  - Uncontrolled / irreproducible
  - Unwanted foaming of multicomponent liquids
- Sparge = blow bubbles
  - Polydisperse or monodisperse
  - Uncontrolled/non-uniform liquid fraction



# Foam production II.

- *in-situ* release / production of gas

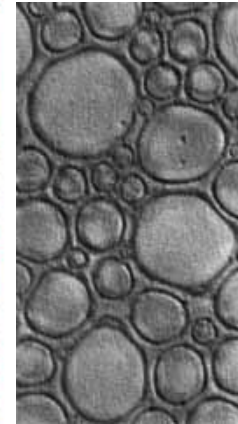
- nucleation

- eg CO<sub>2</sub> in beer



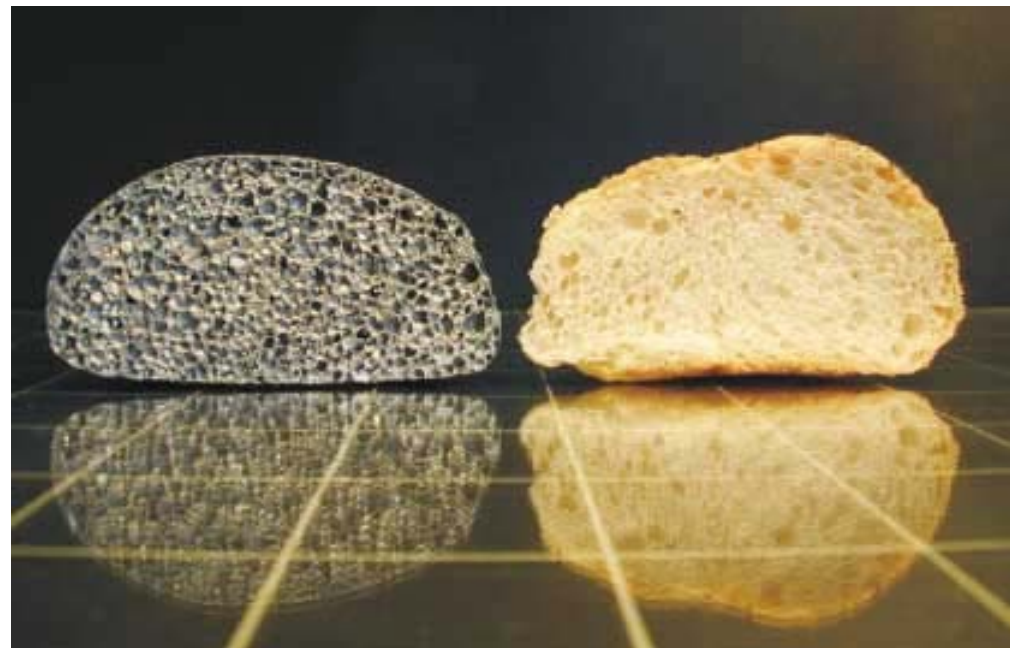
- aerosol:

- eg propane in shaving cream
- small bubbles!



- active:

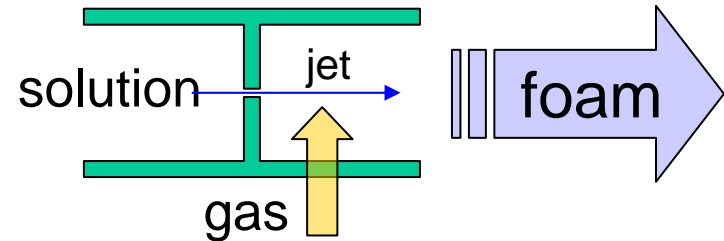
- eg H<sub>2</sub> in molten zinc
- eg CO<sub>2</sub> from yeast in bread



# Foam production III.

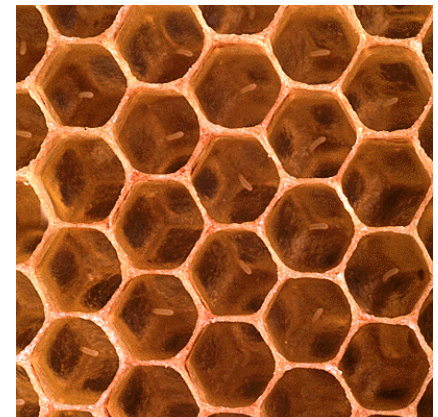
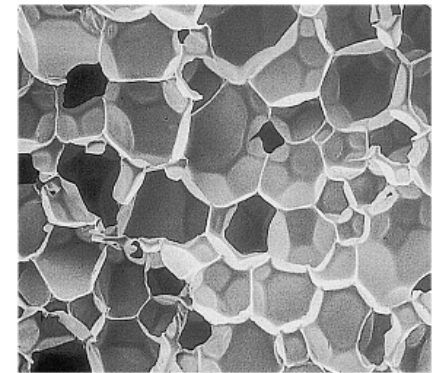
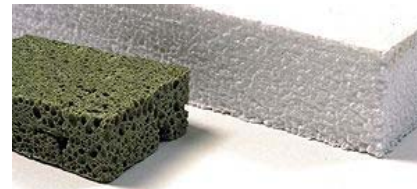
- turbulent mixing of thin liquid jet with gas

- vast quantities
- small polydisperse bubbles
- controlled liquid fraction
  - lab samples
  - firefighting
  - distributing pesticides/dyes/etc.
  - covering landfills
  - supressing dust
  - ...



# *Foam production IV.*

- many materials can be similarly foamed
  - nonaqueous liquids (oil, ferrofluids,...)
  - polymers (styrofoam, polyurethane,...)
  - metals
  - glass
  - concrete
  
- variants found in nature
  - cork
  - bone
  - sponge
  - honeycomb



# *Foams produced by animals*

- spittle bug:



- cuckoo spit / froghoppers:



- stickleback-fish's nest

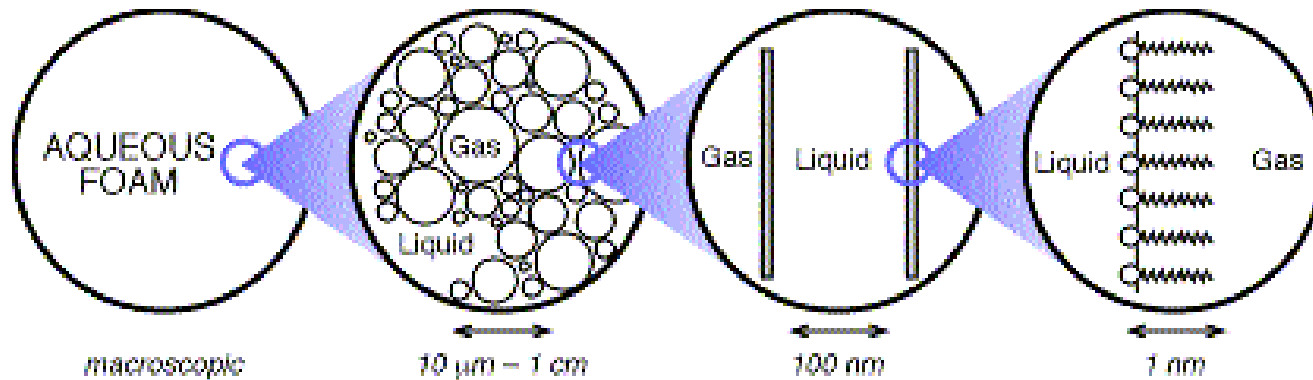
- antifoaming agents
  - prevent foaming or break an existing foam



- mysterious combination of surfactants, oils, particles,...

# *Microscopic behavior*

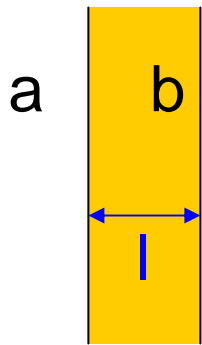
- look at progressively larger length scales...



- surfactant solutions
- soap films
- local equilibrium & topology

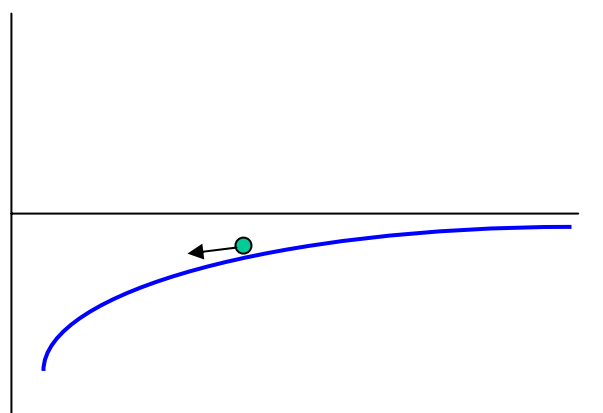


- bubbles quickly coalesce – no foam
  - van der Waals force prefers monotonic dielectric profile; therefore, bubbles attract:



“effective interface potential”  
is free energy cost per unit area:

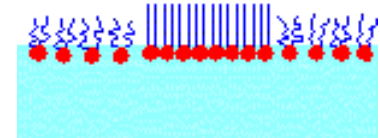
$$V_{\text{vdw}}(l) = -A/12\pi l^2, \quad A = \text{Hamaker constant}$$



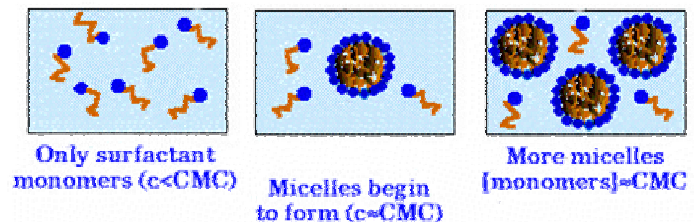
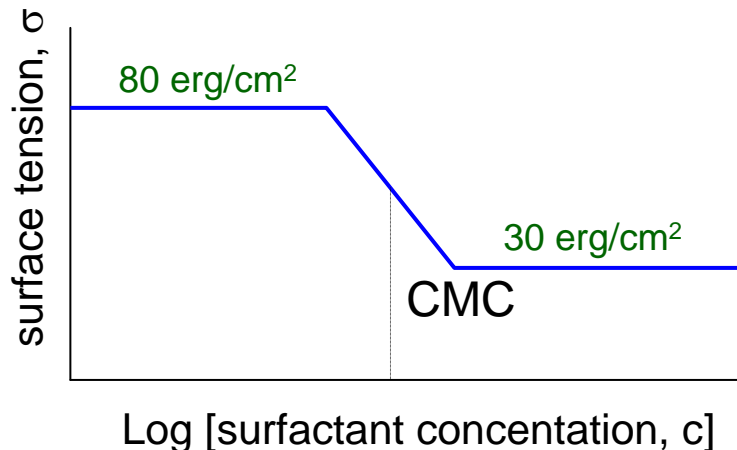
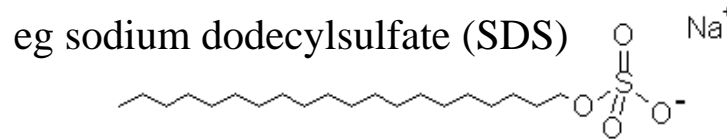
# Surfactant solution

- **surface active agent** – adsorbs at air/water interface

- head: hydrophilic (eg salt)
- tail: hydrophobic (eg hydrocarbon chain)



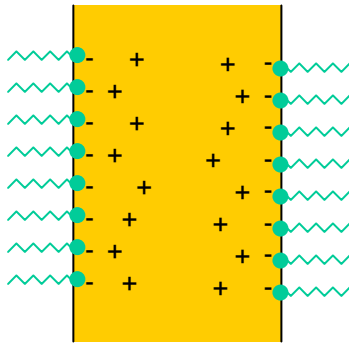
- lore for good foams...
  - chain length: short enough that the surfactant is soluble
  - concentration: just above the “critical micelle concentration”



{NB: lower  $\sigma$  doesn't stabilize the foam..}

# *Electrostatic “double-layer”*

- adsorbed surfactants dissociate, cause repulsion necessary to overcome van der Waals and hence stabilize the foam
  - electrostatic
  - entropic (dominant!)



free energy cost per unit area:

$$V_{DL}(l) = (64k_B T \rho / K_D) \text{Exp}[-K_D l],$$

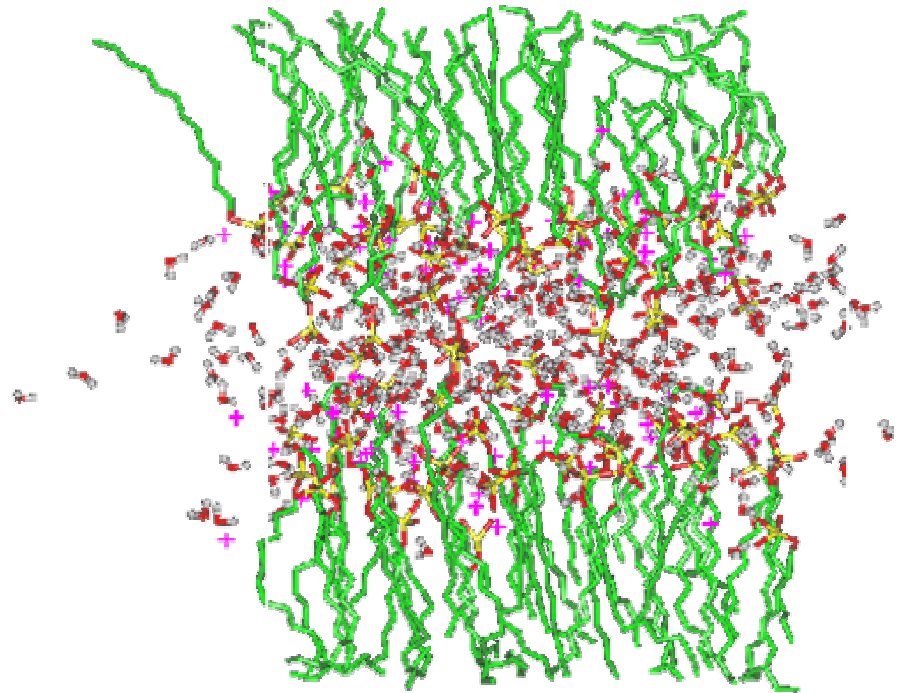
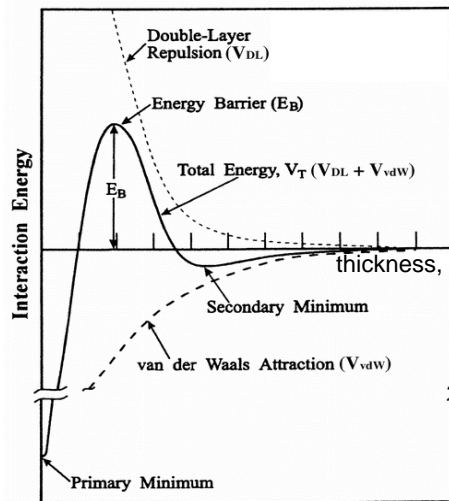
$\rho$  = electrolyte concentration

$K_D^{-1} \sim \rho^{-1/2}$  = Debye screening length

- NB: This is similar to the electrostatic stabilization of colloids

- film tension / interface potential / free energy per area:

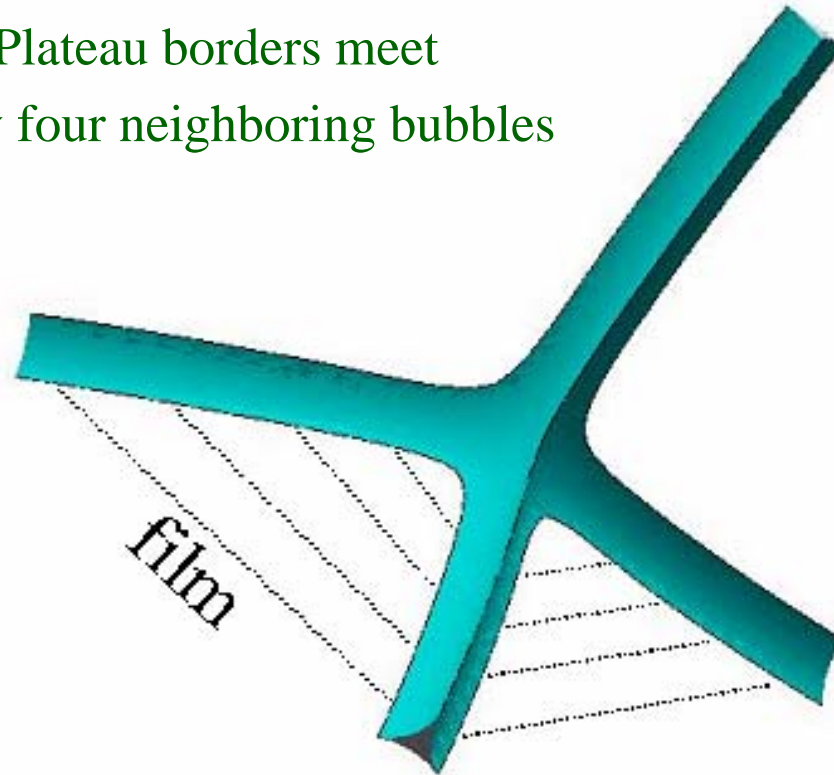
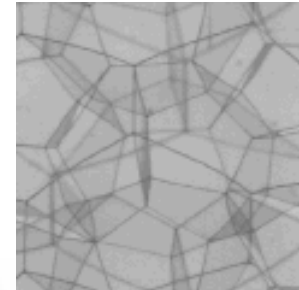
$$\gamma(l) = 2\sigma + V_{\text{VDW}}(l) + V_{\text{DL}}(l) \sim 2\sigma$$



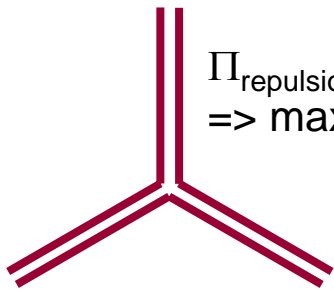
- disjoining pressure:  $\Pi(l) = -d\gamma/dl$

– vanishes at equilibrium thickness,  $l_{\text{eq}} \sim K_{\text{D}}^{-1}$  (30-3000Å)

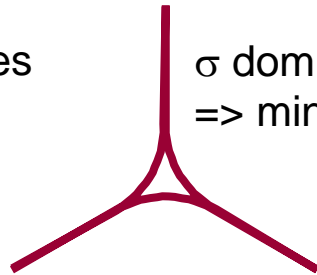
- Plateau border
  - scalloped-triangular channel where three films meet
  - the edge shared by three neighboring bubbles
- Vertex
  - region where four Plateau borders meet
  - the point shared by four neighboring bubbles



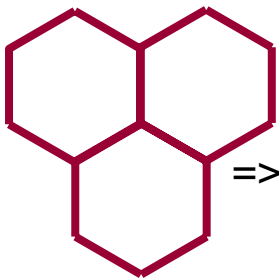
- division of liquid between films-borders-vertices
  - repulsion vs surface tension
  - wet vs dry



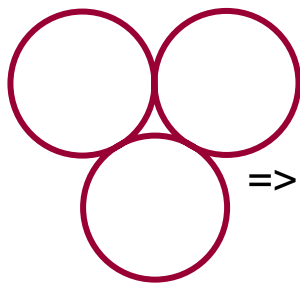
$\Pi_{\text{repulsion}}$  dominates  
=> maximize  $l$



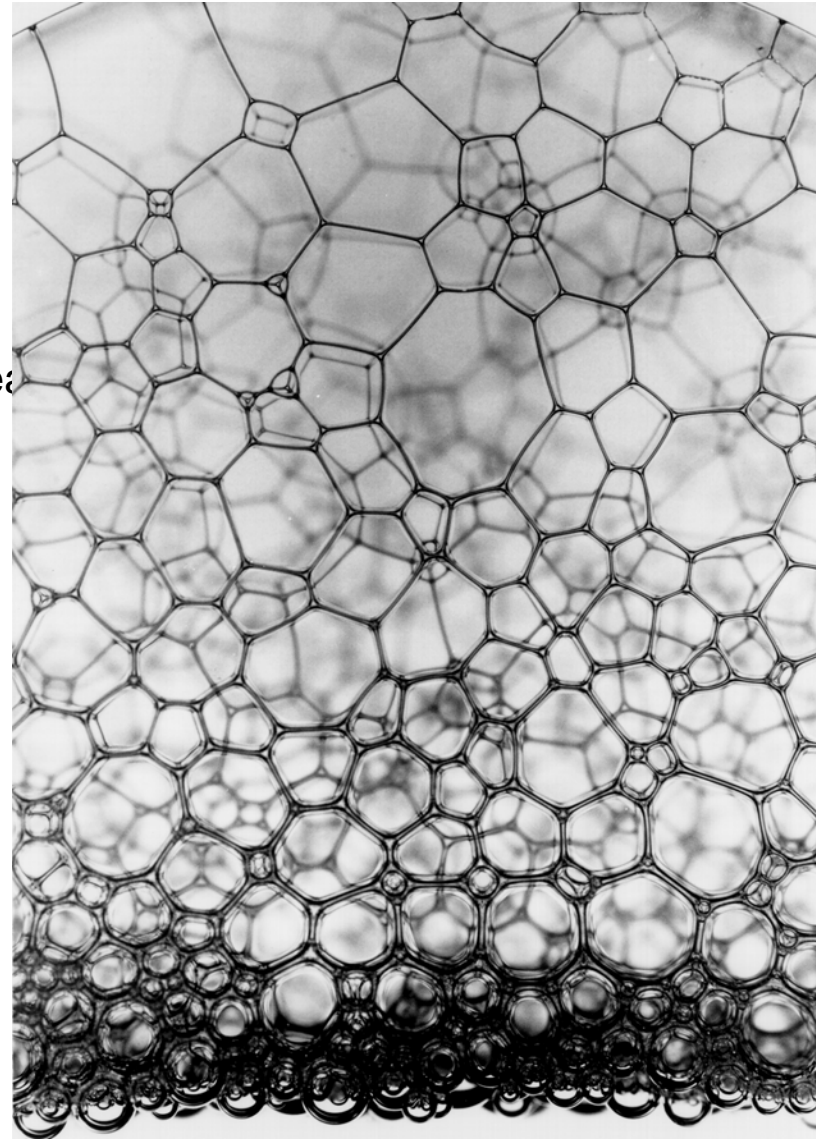
$\sigma$  dominates  
=> minimize area



dry  
=> polyhedral



wet  
=> spherical



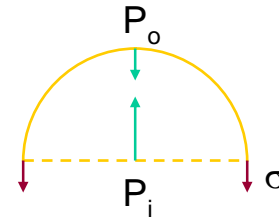
- the pressure is greater on the inside a curved interface

due to surface tension,  $\sigma = \text{energy} / \text{area} = \text{force} / \text{length}$



– forces on half-sphere:

- $\Sigma F_{\text{up}} = P_i \pi r^2 - P_o \pi r^2 - 2\pi \sigma r = 0$

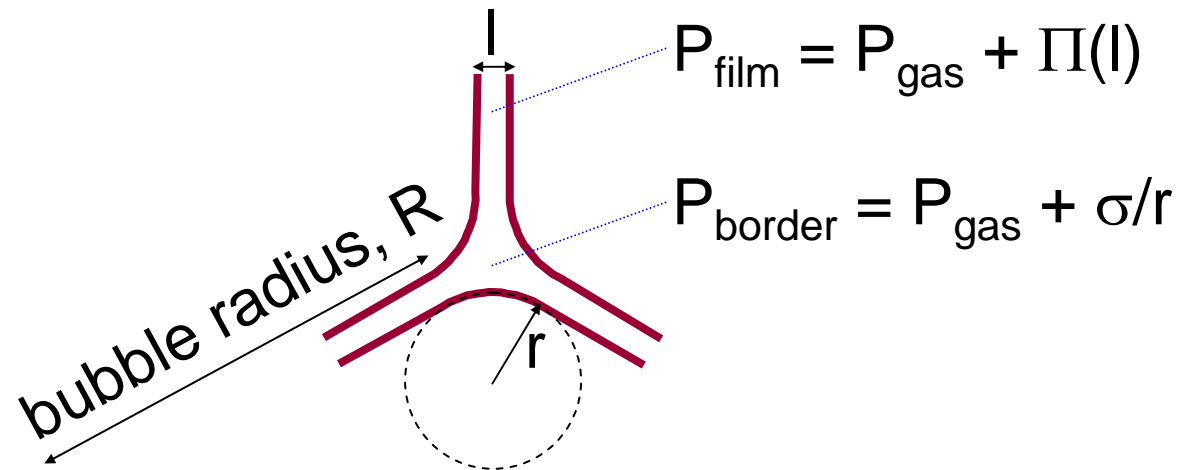


– energy change = pressure x volume change:

- $dU = (\Delta P)4\pi r^2 dr$ , where  $U(r) = 4\pi r^2 \sigma$

# Liquid volume fraction

- liquid redistributes until liquid pressure is same everywhere



- typically: film thickness  $l \ll$  border radius  $r \ll$  bubble radius  $R$ 
  - liquid volume fraction scales as  $\varepsilon \sim (lR^2 + r^2R + r^3)/R^3 \sim (r/R)^2$
  - most of the liquid resides in the Plateau borders
    - PB's scatter light...
    - PB's provide channel for drainage...



# Plateau's rules for dry foams

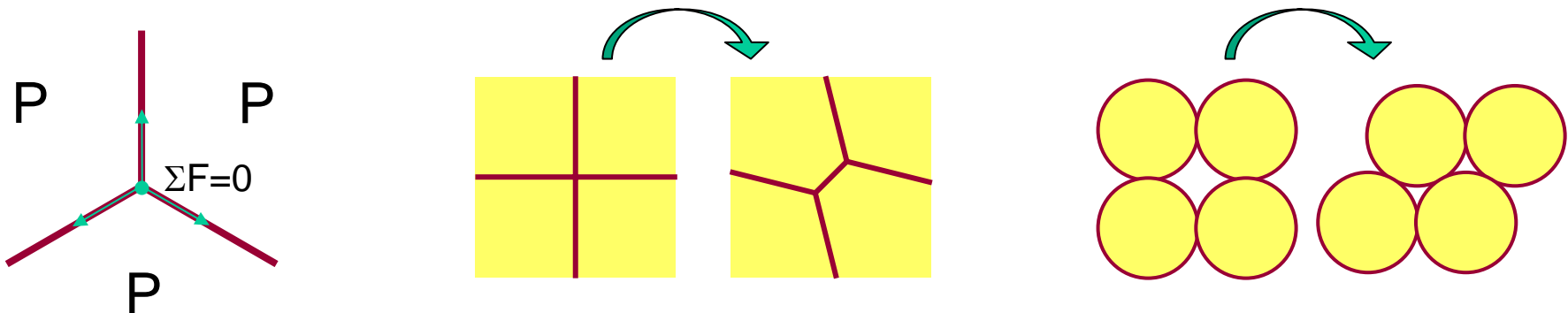
- for mechanical equilibrium:
  - i.e. for zero net force on a Plateau border,
  - zero net force on a vertex,
  - and  $\Sigma\Delta P=0$  going around a closed loop:



(1) films have constant curvature & intersect three at a time at  $120^\circ$

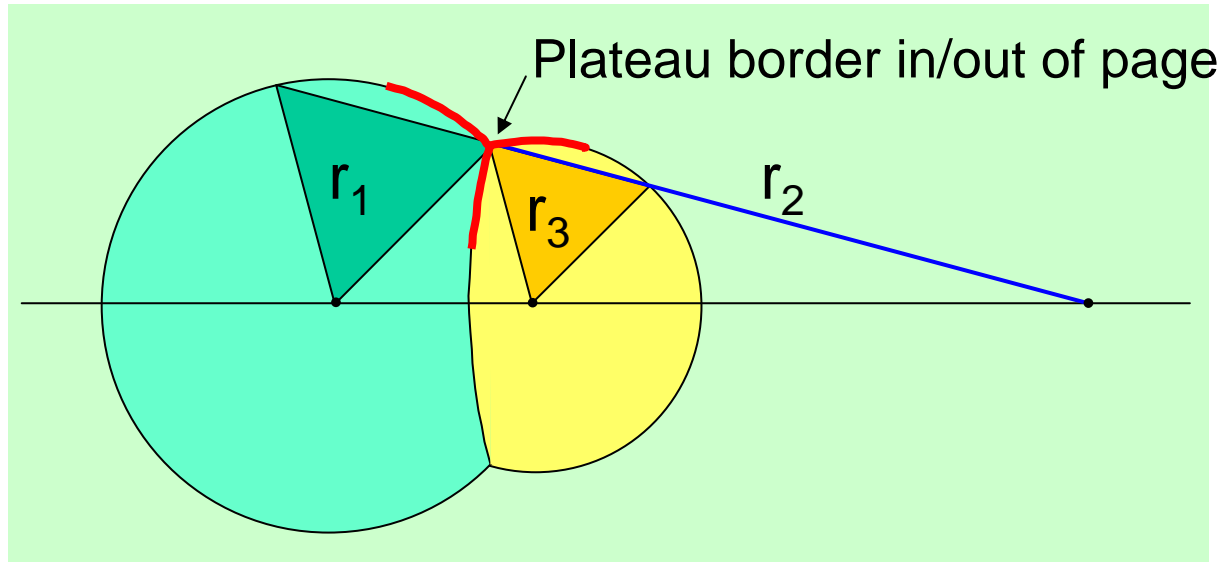
(2) borders intersect four at a time at  $\cos^{-1}(1/3)=109.47^\circ$

- rule #2 follows from rule #1
- both are obviously correct if the films and borders are straight:



# Rule #1 for straight borders

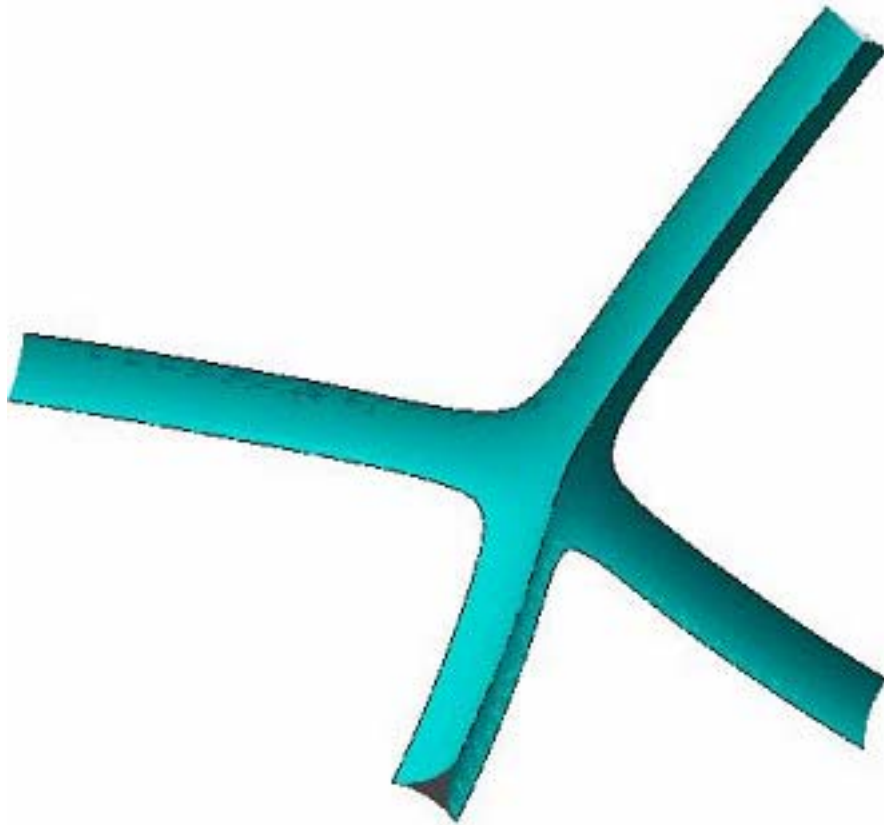
- choose  $r_1$  and orientation of equilateral triangle
- construct  $r_2$  from extension down to axis
- construct  $r_3$  from inscribed equilateral triangle
  - NB: centers are on a line



- films meet at  $120^\circ$  (triangles meet at  $60^\circ$ - $60^\circ$ - $60^\circ$ , and are normal to PB's)
- similar triangles give  $(r_1+r_2)/r_1 = r_2/r_3$ , i.e.  $1/r_1 + 1/r_2 = 1/r_3$  and so  $\Sigma P=0$

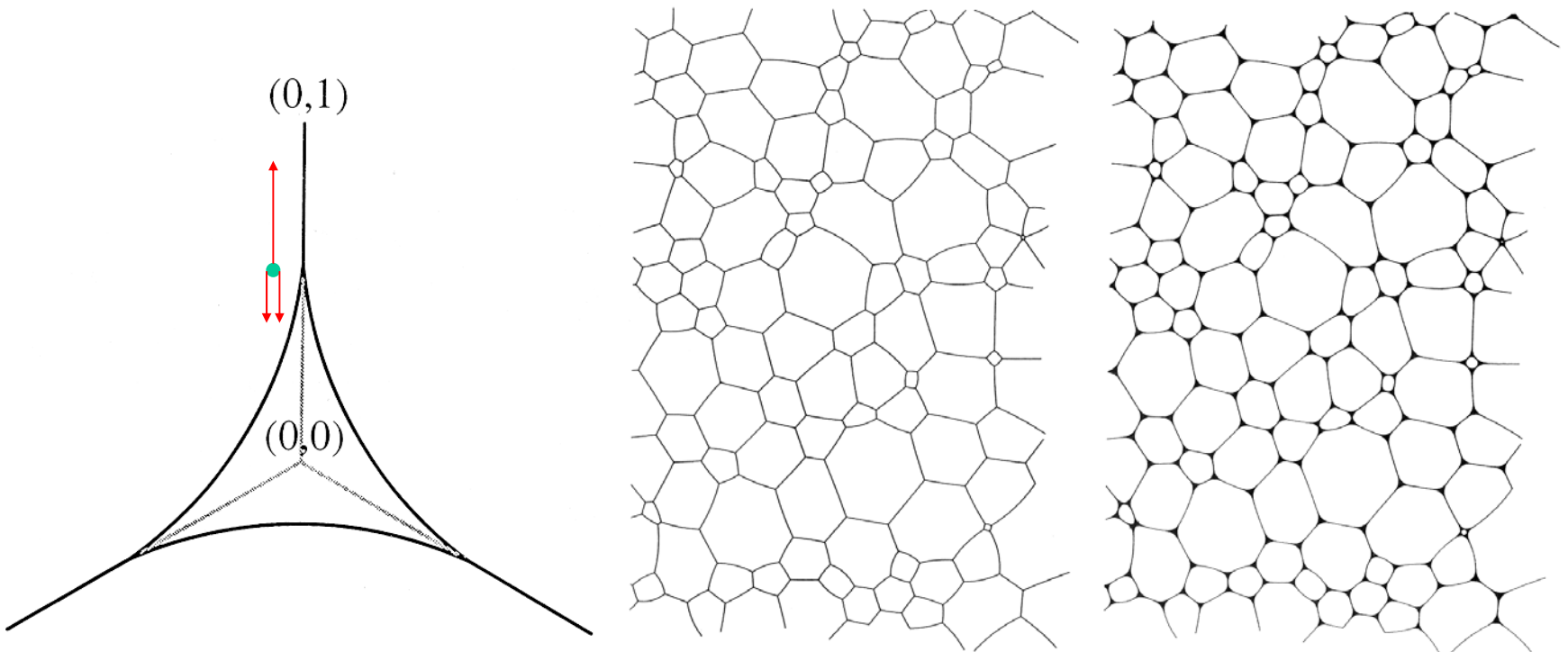
# *Curved Plateau borders*

- proof of Plateau's rules is not obvious!
  - established in 1976 by Jean Taylor



# Decoration theorem for wet foams

- for  $d=2$  dimensions, an equilibrium wet foam can be constructed by *decorating* an equilibrium dry foam
  - can you construct an elementary proof?
- PB's are circular arcs that join tangentially to film
- theorem fails in  $d=3$  due to PB curvature



- periodic foam structures
- disordered foam structures
  - experiment
  - simulation